



BUILDING A  
New Standard  
OF Success

• HMH Educational Services  
**Expressions**  
Common Core

# Math Models

with Math Expressions



HOUGHTON MIFFLIN HARCOURT

# Workshop Objectives



- **Teachers will learn how to use unique manipulatives and models to compose and decompose larger numbers.**
- **Teachers will learn how to use the addition/subtraction/multiplication/division models that support student understanding and acquisition of non-traditional algorithms.**



# Warm Up

The number is less than 1000

The number in the one's place is the only positive even number to have four letters in its spelling

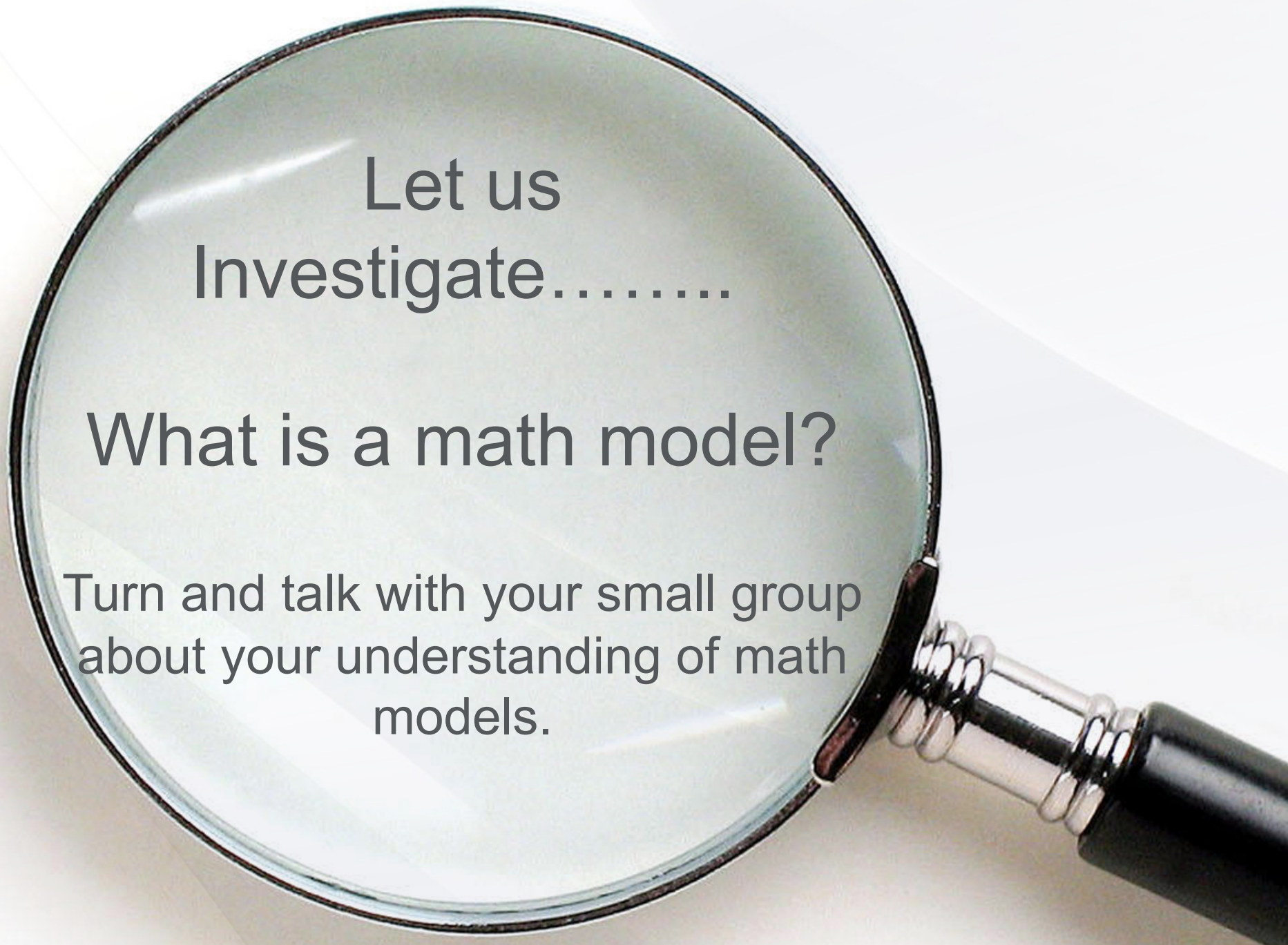
The number in the ten's place is an odd prime number less than 5

The number in the hundred's place is the sum of the digits in the ones and tens place plus 1.

How many calories are in your Grande Honey Spice Latte and Blueberry Muffin?







Let us  
Investigate.....

What is a math model?

Turn and talk with your small group  
about your understanding of math  
models.



**Math Mountain**



**Math  
Mountain**



Houghton Mifflin Harcourt

# Math Mountain

6

Math  
Mountain



# Math Mountains

First Grade

$$7 + 5 = \square$$

3 2

$$10 + 2 = 12$$

Second Grade

$$60 - \square = 29$$

$$\begin{array}{r} 60 \\ - \quad \quad - \\ \hline 31 \quad + \quad 29 \end{array}$$





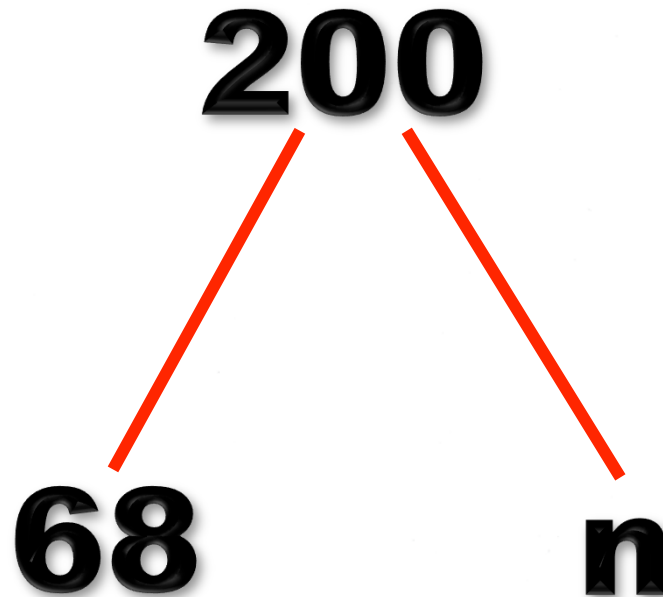


## Complex Problems

**Last night, 68 people attended the early showing of a theater movie. How many people attended the late showing if the total attendance for both showings was 200 people?**



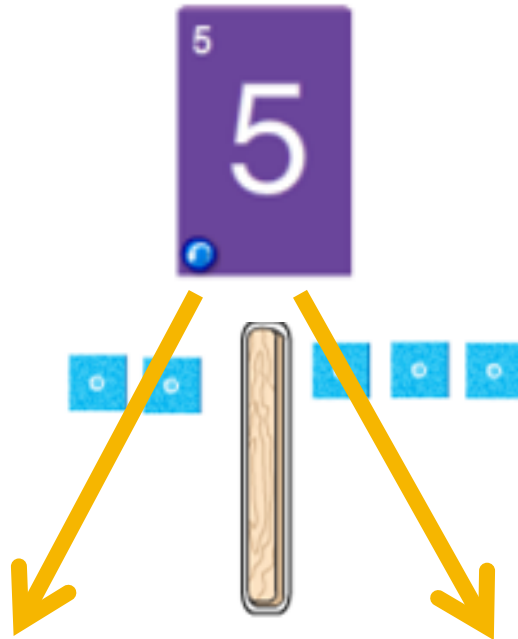
# Complex Problems



Last night, 68 people attended the early showing of a theater movie. How many people attended the late showing if the total attendance for both showings was 200 people?



# Creating a Math Model





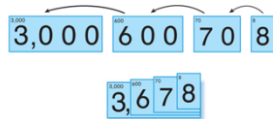
# Secret Code Cards

## Place Value Concepts

Lessons

1 2 4

**Whole Number Secret Code Cards** Students explore place value by assembling Secret Code Cards to form multidigit numbers. The cards show place values. To make the number 3,678, students select the cards representing 3 thousands, 6 hundreds, 7 tens, and 8 ones.

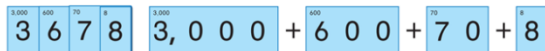


Using the cards is beneficial for students because the card shows how the position of the digit in the number determines the digit. For example, with the cards students can see more that 3 on the thousands card is 3,000 while a 3 on the tens card is 30.

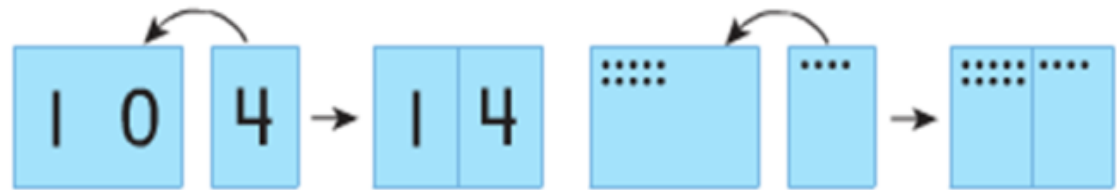
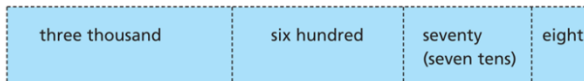
The uniform use of the cards to show the standard, expanded, and word form of numbers allows students to make connections between the different forms.

### Standard Form

### Expanded Form



**Word Form** The backs of the cards show the values in word form.

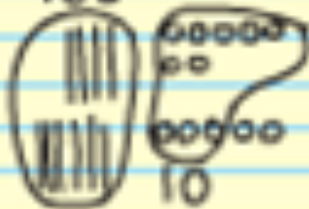


# Quick Draws and Proof Pictures

47 coyotes hunted for food at night.  
65 lizards hunted for food during the day.  
How many desert animals hunted for food?

$$\begin{array}{r} 47 \\ + 65 \\ \hline 100 \\ + 12 \\ \hline 112 \end{array}$$

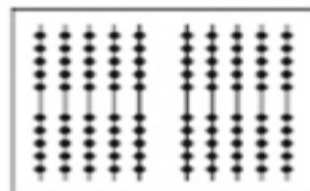
100



10

112 desert animals hunted for food.

## Meanings for Drawings



Ten tens and one  
hundred ones



One ten is  
ten ones.

## MATH DRAWINGS (SKETCHES)



Three hundreds seven tens nine ones

## QUANTITY WORDS



# 2 Digit Addition

**Grade 1: Unit 4 (Sets up for proofing)**

**Unit 8: Explores 2-Digit Addition**

**Grade 2: Unit 6**

**Grade 3: Unit 4**

**Grade 4: Unit 1**

**Grade 5: Unit 2 (Decimals)**

**Grade 6: Unit 3 completes all computation**





50	
5	8

30	
3	6

$$\begin{array}{r} 58 \\ + 36 \\ \hline \end{array}$$

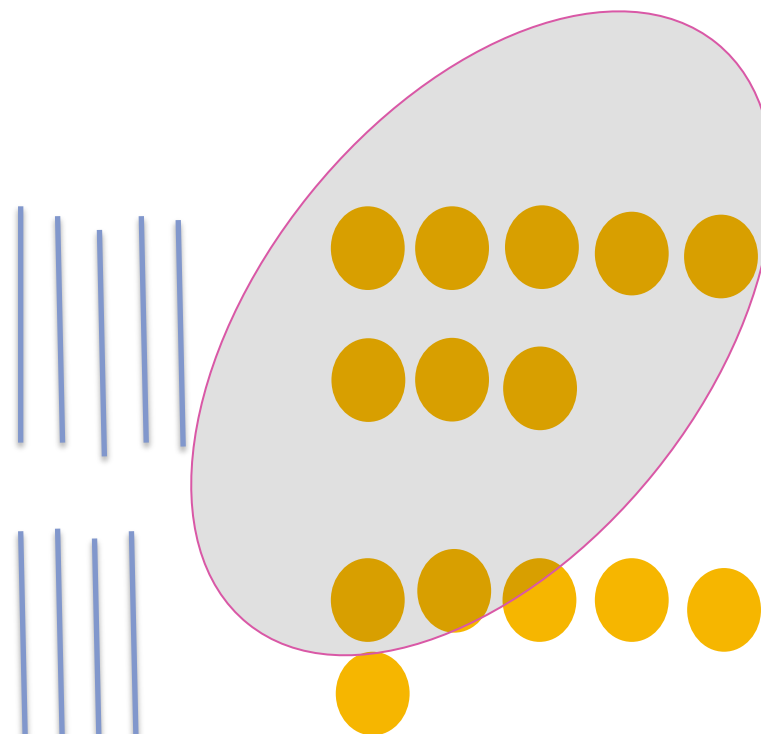
$$\begin{array}{r} 80 \\ 14 \\ \hline \end{array}$$

Show all Subtotals

$$94$$

# Proof Drawing

$$\begin{array}{r} 58 \\ + 36 \\ \hline \end{array}$$



50	
5	8

30	
3	6

New Groups Above

● ● ● ● ●	
● ● ●	
	1
	58

● ● ● ● ●	
● ● ●	
+	36

	1
<hr/>	
	94

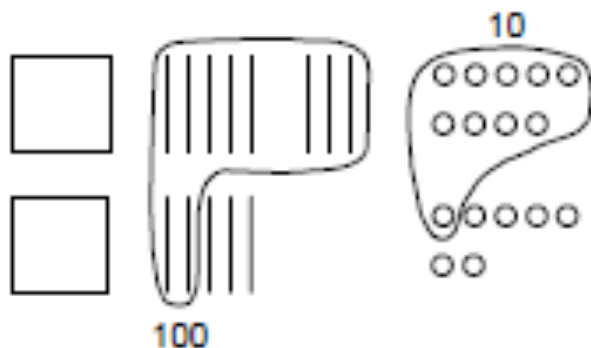
New Groups Below





$$69 + 25 =$$

Quantity Model



New Groups  
Below

$$\begin{array}{r} 189 \\ + 157 \\ \hline 346 \end{array}$$

Write All  
Totals

$$\begin{array}{r} 189 \\ + 157 \\ \hline 200 \\ 130 \\ 16 \\ \hline 346 \end{array}$$

Current Common  
New Groups  
Above

$$\begin{array}{r} 1 \quad 1 \\ 189 \\ + 157 \\ \hline 346 \end{array}$$



# Subtraction

**Grade 2: Unit 4**

**Grade 3: Unit 4**

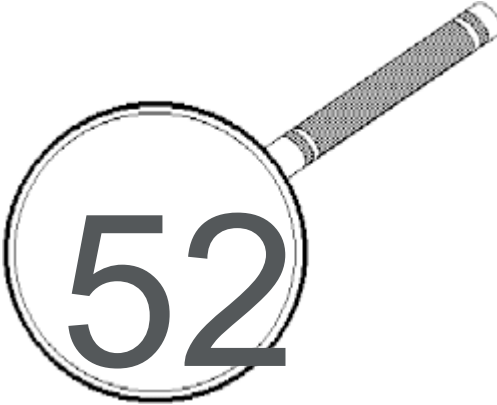
**Grade 4: Unit 1**

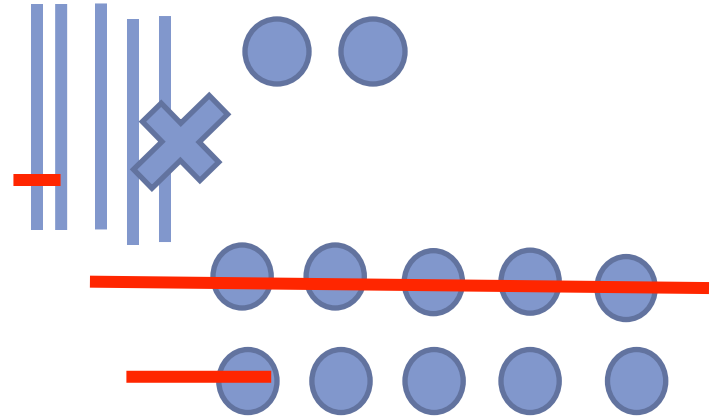
**Grade 5: Unit 2 (Decimals)**

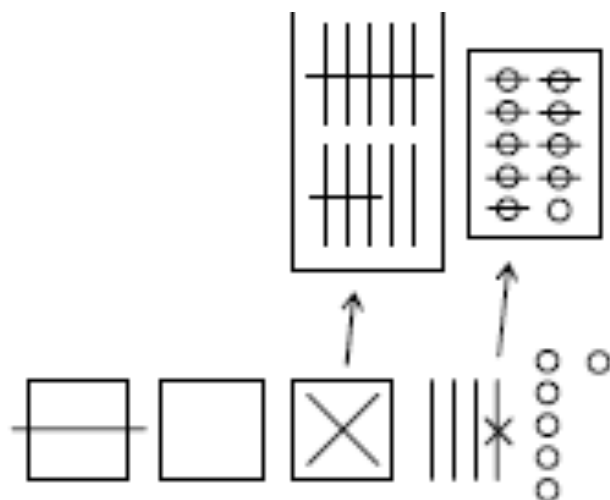
**Grade 6: Unit 3 (Decimals)**



# Subtraction


$$\begin{array}{r} 52 \\ -16 \\ \hline \end{array}$$





Ungroup Everywhere First,  
Then Subtract Everywhere

Left → Right      Right → Left

$$\begin{array}{r} 13 \\ 2 \cancel{14} 16 \\ \underline{- 189} \\ 157 \end{array}$$

$$\begin{array}{r} 13 \\ 2 \cancel{3} 16 \\ \underline{- 189} \\ 157 \end{array}$$

R → L Ungroup,  
Then Subtract,  
Ungroup, Then  
Subtract

$$\begin{array}{r} 13 \\ 2 \cancel{3} 16 \\ \underline{- 189} \\ 157 \end{array}$$



# Multiplication

**Grade 2: Unit 4 (sets up equal groups)**

**Grade 3: Units 1 and 2**

**Grade 4: Unit 2**

**Grade 5: Unit 4 (decimals)**

**Grade 6: Unit 3**



$$\begin{array}{r} 2 \\ 13 \\ \times 7 \\ \hline 91 \end{array}$$

10

7

70

3

21

$$\begin{array}{r} 70 \\ +21 \\ \hline 91 \end{array}$$





$$\begin{array}{r} \phantom{0}^2 17 \\ \times 14 \\ \hline \phantom{0} 68 \\ 170 \\ \hline 238 \end{array}$$

$$\begin{array}{r} 17 \\ \times 14 \\ \hline 68 \\ 170 \\ \hline 238 \end{array}$$

10

4

17



$$\begin{array}{r}
 17 \\
 \times 14 \\
 \hline
 68 \\
 170 \\
 \hline
 238
 \end{array}$$

10

4

17
170
68
238





Area Model	Rectangle Sections	Expanded Notation	Short-Cut
<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 60 \\ + 7 \\ \hline \end{array}</math> </div> <div style="border: 1px solid black; padding: 10px; display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>40</span> <span>+ 3</span> </div> <div style="display: flex; justify-content: space-between; width: 100%;"> <div style="border-right: 1px solid black; padding: 5px; text-align: center;">2400</div> <div style="border-right: 1px solid black; padding: 5px; text-align: center;">180</div> </div> <div style="display: flex; justify-content: space-between; width: 100%;"> <div style="border-right: 1px solid black; padding: 5px; text-align: center;">280</div> <div style="padding: 5px; text-align: center;">21</div> </div> </div> </div>	$\begin{array}{r} 2400 \\ 180 \\ 280 \\ + \quad 21 \\ \hline 2881 \end{array}$	$\begin{array}{r} 43 = 40 + 3 \\ \times 67 = 60 + 7 \\ \hline 60 \times 40 = 2400 \\ 60 \times 3 = 180 \\ 7 \times 40 = 280 \\ 7 \times 3 = 21 \\ \hline 2881 \end{array}$	$\begin{array}{r} 1 \\ 2 \\ 43 \\ \times 67 \\ \hline 301 \\ 258 \\ \hline 2881 \end{array}$

# Division

**Grade 3: Related to Multiplication  
Unit 2**

**Grade 4: Unit 3, Lessons 2 and 3**

**Grade 5: Unit 4 (Decimals)**



$$326 = 300 + 20 + 6$$

$3 \times 300 = 900$	$3 \times 20 = 60$	$3 \times 6 = 18$
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$$\begin{array}{r} 900 \\ 60 \\ + 18 \\ \hline 978 \end{array}$$

$$\underline{300} + \underline{20} + \underline{6} = 326$$

$\begin{array}{r} 978 \\ - 900 \\ \hline 78 \end{array}$	$\begin{array}{r} 78 \\ - 60 \\ \hline 18 \end{array}$	$\begin{array}{r} 18 \\ 18 \\ \hline 0 \end{array}$
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# Rectangle Sections

$$\begin{array}{r}
 40 \quad + \quad 3 \quad = 43 \\
 \hline
 2881 \\
 - 2680 \\
 \hline
 201
 \end{array}
 \quad
 \begin{array}{r}
 201 \\
 \hline
 201 \\
 \hline
 0
 \end{array}$$

# Expanded Notation

$$\begin{array}{r}
 3 \quad 43 \\
 40 \quad ] \\
 \hline
 67 \overline{) 2881} \\
 - 2680 \\
 \hline
 201 \\
 - 201 \\
 \hline
 \hline
 \end{array}$$

# Digit by Digit

$$\begin{array}{r}
 43 \\
 67 \overline{) 2881} \\
 - 268 \\
 \hline
 201 \\
 - 201 \\
 \hline
 \hline
 \end{array}$$



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